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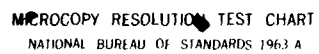
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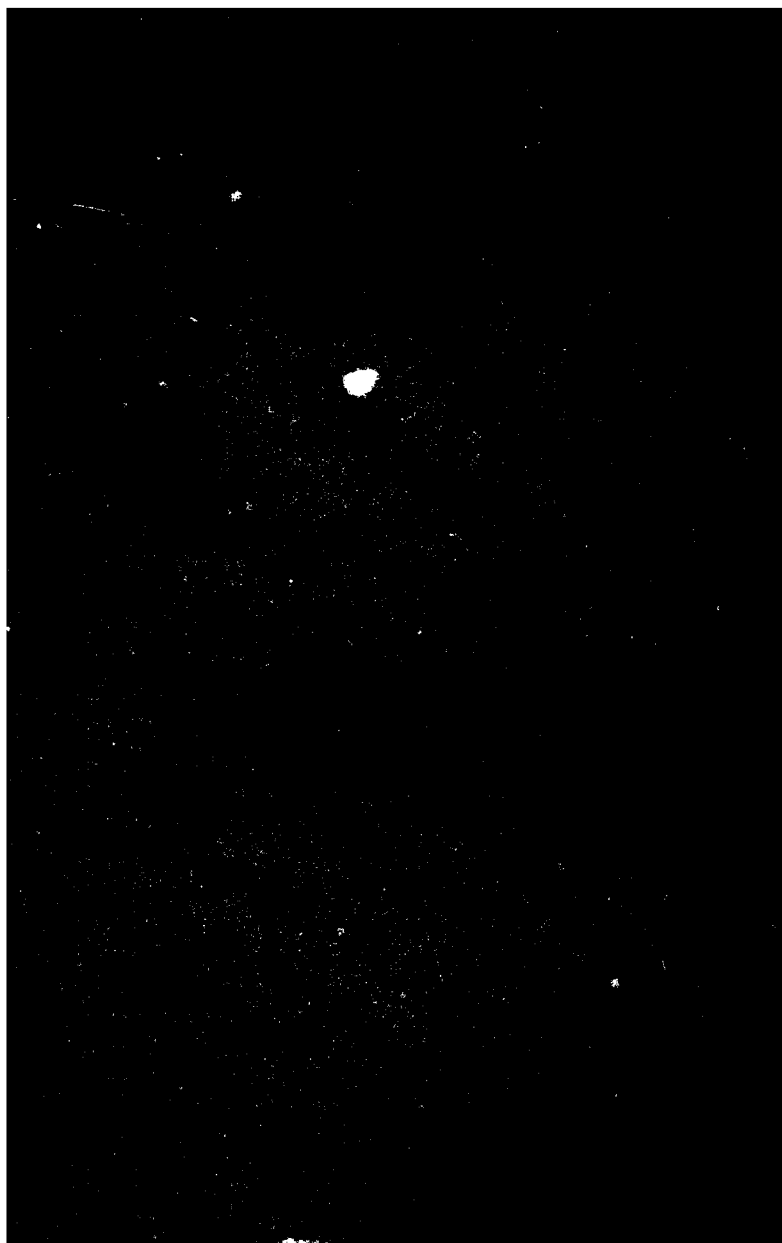

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A Dynamic Econometric Retention Model (DERM) is designed for studying the effects of alternative compensation policies on the retention behavior of Air Force officers, including the Uniformed Services Retirement Modernization Act, the President's Commission on Military Compensation, and the Uniformed Services Retirement Benefits Act. DERM is a model of sequential behavior containing the appropriate econometric method for estimating the retention rate. The econometric method is a maximum likelihood procedure endogenously determined by the specification of the behavioral model. It differs from earlier approaches in that it explicitly considers the *behavioral* effects flowing from decomposing the disturbance term into permanent and transitory components. An important implication of DERM is that retention rates depend both on prospective future returns to remaining in the military on past occurrences. If this is correct, then simple regression models should overpredict the retention gains of proposed compensation policies, exactly what happens in two recent reenlistment studies that use regression analysis. 38 pp. Ref. (Author)

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June 1980

# Estimating Military Personnel Retention Rates: Theory and Statistical Method

Glenn A. Gotz, John J. McCall

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A Project AIR FORCE report  
prepared for the  
United States Air Force



PREFACE

As the cost of defense manpower has increased, the Congress and the Executive Branch have examined various elements of the military personnel and compensation systems. Since 1969 at least six major revisions to the military nondisability retirement system have been proposed and, more recently, there have been substantial changes suggested for the percentage distribution of officers across grades.

The evaluation of alternative personnel management and compensation systems is necessarily incomplete if it does not account for changed incentives, hence changed retention patterns, among those subject to the revised systems. This report develops a decision model from which it will be possible to predict expected retention patterns of officers under alternative systems. The model is a dynamic programming model that examines in a unified manner both the pecuniary and non-pecuniary incentives to remain in or leave the Air Force. It accounts for persistent differences in tastes and opportunities among officers and for transient factors that may alter retention decisions. The final stage of Rand's research on officer retention behavior will be to statistically estimate this model's parameters and to examine the retention, personnel force structure, and cost implications of alternative personnel and compensation policies.

This report was prepared for the Deputy Chief of Staff, Manpower and Personnel, Headquarters, United States Air Force, under the Project AIR FORCE project "Officer Personnel Management Study."

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### SUMMARY

This report develops a dynamic programming decision model from which it will be possible to calculate, for alternative personnel and compensation policies, the probability that an Air Force officer will remain in the service. The model accounts for persistent differences in tastes and opportunities among officers and for transient factors that may alter retention decisions. It also includes the most important institutional factors affecting an officer's career: promotion probabilities and timing, regular force integration probabilities, and mandatory separation and retirement probabilities. It unites the officer's taste for the service and expectations of transient factors with income for each potential combination of future grade and year of service and expected civilian income opportunities.

Implications for the retention behavior of service members drawn from this new model can diverge substantially from simple regression model predictions, especially in the implications of alternative retirement systems. The intellectual development of the model is presented in three stages so that the reader can understand the sources of these differences. First is a description of the stochastic dynamic program developed by Gotz (forthcoming) and by Gotz and McCall (1979). A statistical model developed by Heckman and Willis (1976) is appended to this model to account for population heterogeneity in tastes and for transient disturbances. This statistical model is known here as the Preliminary Econometric Method (PEM). Finally, the Dynamic Econometric Retention Model (DERM) is a sequential model of economic behavior that contains within it the appropriate econometric method for estimating the retention rate (the fraction of those eligible to remain in the service who do remain for at least one more year). The procedure is novel in that the econometric method is endogenously determined by the specifications of the stochastic economic model.

Although DERM is a more formidable mathematical model than simple regression models, its behavioral implications are more intuitively plausible. For example, in the years following an officer's receipt

of a sizable bonus payment with no associated active duty service obligation, a regression model would predict no change in retention rates as a consequence of the bonus. One might expect, however, that those induced by the bonus to remain in the service would not remain beyond the bonus year of service with the same frequency as those who would have remained even in the absence of the bonus. DERM (and PEM) predicts that for those induced to remain by the bonus, the retention rate for the year after the bonus would be lower than the retention rate of those who would have remained without the bonus.

In DERM, retention rates depend both on prospective future returns to remaining in the military and on past occurrences. If that is correct, then DERM implies certain observable errors in regression model results; the reenlistment studies by Chipman and Mumm (1978) and Warner (1978) indeed contain these errors, which imply that regression models will overpredict the expected man-year gains to such retirement system alternatives as those proposed by the President's Commission on Military Compensation and the Uniformed Services Retirement Benefits Act.

Not uncommon to proposed retirement system alternatives is some form of "grandfathering" members of the current force when the new system is instituted. The decisions of service members to serve under the old or the new system can determine the structure of the personnel force for many years. Such predictions flow naturally from the structure of DERM.

ACKNOWLEDGMENTS

The authors are indebted to Nicolas Kiefer of the University of Chicago and G. S. Maddala of the University of Florida, and to Rand colleagues Peter Stan and Michael Ward for their valuable comments.

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## I. INTRODUCTION

This report presents a conceptual model of the individual's decision to remain in or to leave the military. Although we examine Air Force officers, with only minor modifications the model will apply to the enlisted and officer forces of each service.\* The essence of the approach is a blending of two literatures: the economics of uncertainty and the econometric modeling of longitudinal data.

Since 1969 there have been at least six major proposals for revising the military's nondisability retirement system: The First Quadrennial Review of Military Compensation was followed by the proposals of the Interagency Committee, the Uniformed Services Retirement Modernization Act, the Defense Manpower Commission, the President's Commission on Military Compensation; and the latest is the Uniformed Services Retirement Benefits Act by the Office of the Secretary of Defense. The proposed changes have ranged from fairly modest reductions in the value of retirement annuities to restructuring of the timing and value of vesting privileges.

Our research is an attempt to evaluate these proposals. All of the proposed changes to the retirement system lack knowledge of how these changes will affect the costs and capabilities of the various components of the military forces. It is difficult enough to assess the relative capabilities of personnel force structures when those structures are known for sure; consider the task of evaluating alternative retirement plans when their effects on retention patterns and, hence, force structures and costs are unknown. The retention analyses associated with the proposed Uniformed Services Retirement Benefits Act have been clearly superior to previous efforts, but the tools have not been adequate to the task even in those studies.

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\* A model like this also can be used to study quits and retirements in the private sector. An extended version would encompass the contractual model of matchmaking as presented in the work of Mortensen (1978) and Diamond and Maskin (1979).

### BACKGROUND

The last few years have seen many contributions to the economics of uncertainty,<sup>\*</sup> and most economists conclude that probabilistic considerations are important in the formulation of economic models. Further, new econometric techniques have been designed for analyzing the recently available panel data.<sup>†</sup> One of the main points demonstrated in the theory literature is that the behavior of economic actors depends on the structure of the stochastic environment. Optimal policies for firms and consumers cannot be calculated without specifying the underlying stochastic process. Econometric modeling of longitudinal data has also considered the composition of the stochastic disturbance, usually to achieve more efficient estimation procedures. For example, the error term may be seen as emanating from two sources--one caused by the heterogeneity of the economic actors and the other by transient fluctuations. The decomposition of the error term into persistent and transient factors has become increasingly attractive with the advent of panel data.

The economics of uncertainty and econometric analysis of panel data are two of the most important recent developments in economics. These two strands have obviously influenced one another, but they have yet to be combined in the analysis of any problem. The probabilistic elements to which theory assigns importance have not explicitly entered into the econometric modeling and empirical testing. Indeed, in the empirical literature in this field, there is sometimes only a vague correspondence between the theoretical model and the empirical testing.<sup>‡</sup>

The absence of a close relationship between theory and testing has several causes. Data limitations probably account for most of the discrepancy. Most economic theories are designed with the individual in mind, whereas only aggregate data are available. Thus, an aggregation problem must be resolved before empirical testing can be justified. Furthermore, economic models are sometimes required for explaining

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<sup>\*</sup> For a recent survey, see Lippman and McCall (1980).

<sup>†</sup> See Heckman (1978a).

<sup>‡</sup> For an exception see Kiefer and Neuman (1979).

individual behavior over time. This necessitates a life cycle model, a dynamic model in which the effects of the future are optimally incorporated in present decisionmaking. Such models are frequently difficult to construct; and even when these theoretical obstacles are overcome, the data against which they are measured are not appropriate. They may consist of aggregate time series, aggregate cross sections, and individual cross sections. Only in some cases are the required panel data available. In other cases the researcher has access to the appropriate panel data but uses a one-period model to generate hypotheses, because it is not feasible to build a full-fledged dynamic model.

#### PLAN OF THE REPORT

The purpose of this report is to develop the appropriate model of behavior (and the statistical technique for estimating it) for individuals who are making sequential decisions in an uncertain environment. The distinguishing feature of this model is that differences among individuals are reflected in their behavior rather than being swept into the error term.

The econometric method used to estimate a dynamic model of retirement from the military to the civilian sector, instead of being an appendage, is derived from the theory of sequential behavior. The model is unique in that we have access to the longitudinal data that are necessary for testing; and furthermore, its simple structure permits a detailed characterization of the sequential decision process.

The problem is to choose the optimal time to retire from the military when the objective is to maximize the expected present value of pecuniary and nonpecuniary returns. The problem can be characterized as a stochastic dynamic program. This characterization forms the basis for the subsequent estimation procedures. A preliminary test of the model's adequacy compares actual retirements with the optimal policy of a maximizer of the expected present value of income.

Section III develops a Preliminary Econometric Method (PEM) for estimating the retention rate (the fraction of those eligible to remain in the military who do remain for at least one more year). This

method distinguishes between persistent and transient components of the error term and derives maximum likelihood estimates of the key retention parameters. In PEM, the effects of heterogeneity and transience on the individual's optimal retirement policy are not considered directly. Instead, this method calculates the cost of leaving from a retirement model that ignores both heterogeneity and transience. From the observed sequence of stay-leave decisions, it then estimates the parameters of the (misspecified) stochastic process generating the "true" costs of leaving. The significance of heterogeneity and transience turns on these estimates. This procedure is incompatible with optimal sequential decisionmaking. Heterogeneity and transience enter the scene much earlier and directly affect the calculation of the optimal retirement policy. Their estimation should occur within the optimization setting--the estimation should be imbedded in the dynamic program.

Section IV presents the Dynamic Econometric Retention Model (DERM), which is a model of sequential behavior that includes the appropriate econometric method for estimating the retention rate. The econometric method, a maximum likelihood procedure, is endogenously determined by the specification of the behavioral model. It is basically different from PEM in that it explicitly considers the *behavioral* effects that follow from the decomposition of the disturbance term into permanent and transitory components. That is, the estimation procedure assumes that each individual knows both his permanent component and the distribution of the transitory component. This knowledge influences the calculation of the optimal retention policy and must be explicitly recognized by the econometric method used to estimate the retention parameters.

Both PEM and the estimation procedure of DERM<sup>\*</sup> differ from a simple regression model that completely ignores heterogeneity. The difference is important and has received much attention in the recent econometric

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<sup>\*</sup>Throughout the report the acronym DERM refers to both the model of sequential behavior and the associated maximum likelihood estimation procedure. Proper interpretation of DERM should be clear from the context.



literature on selectivity. As a simple illustration, assume that under current military compensation policies there is a set of retention rates by year of service. Suppose a sizable bonus, say \$10,000, is introduced at some year of service. The only eligibility requirement for the bonus is to have completed the requisite years of service. There is no obligation to continue after receiving the bonus. Each of the three models--the regression model, PEM, and DERM--will predict an increase in the retention rate for each year of service that is less than the year in which the bonus is received. The regression model will predict that retention rates after the bonus year will be the same as before the institution of the bonus. Of course, there will be more people in those later years of service. One might expect, however, that those who were induced to remain in the service by the bonus would not remain beyond the bonus year with the same frequency as those who would have remained even in the absence of the bonus. PEM and DERM are capable of addressing precisely this point. Each predicts that for years after the bonus year, those who were induced to remain by the bonus would have lower retention rates than those who would have remained without the bonus.

In the final section, implications for behavior are drawn from DERM and compared with those of PEM and regression models. In particular, we show that DERM implies certain observable errors in regression model results, and we cite studies that contain those errors. DERM's power is also illustrated when we examine a policy change wherein service members may choose to be "grandfathered" under the old policy or serve under the new one.

We conclude this introduction with a historical note that should clarify the relationship between the stochastic dynamic program of Section II and the econometric methods of Sections III and IV.

The order of presentation corresponds exactly to DERM's historical development. With a stochastic dynamic programming model of individual retirement behavior and assuming that individuals maximize expected present value of future income, we calculated the optimal time to retire for each group of Air Force officers. The groups were chosen so

that on the basis of observed characteristics the members of each group are homogeneous.\*

At this point there is no statistical problem. The model assumes that all relevant characteristics are known, and from this information it is possible to calculate an optimal time to leave. An individual remains in the Air Force until the cost of leaving becomes negative. If observed behavior is in accord with these predictions, the model is judged to be correct.

Alas, life is not so easy. Unobserved differences produce heterogeneity among Air Force officers and affect their retirement decisions. Transient factors also influence retirement decisions. Our basic informational assumption is that although we do not know the heterogeneous factor, the officer does; furthermore, the officer knows the distribution of the transient factor.

To estimate the retention rate for a particular group of Air Force officers at a specified point in their career, econometricians must know the distributions of both the heterogeneous and the transient factors. This knowledge is achieved through statistical estimation. The Preliminary Econometric Method takes cognizance of these factors by adding to the cost of leaving, derived from the stochastic dynamic program, one random variable generating heterogeneity and another generating transience. Each officer solves his dynamic program including *only* those data that are observed by the econometrician and from them his cost-of-leaving function is calculated. Individual differences (a random variable with distribution G) and transience (a random variable with distribution F) are appended to his cost. PEM is a maximum likelihood procedure for estimating the parameters of F and G. Once these estimates are obtained, the econometrician can estimate the retention rate.

But if an officer behaves according to a sequential decision paradigm (namely, dynamic programming) then factors unobserved by the econometrician but known to the officer will be included in his dynamic

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\*This is not literally true, but for all practical purposes, the within-group differences can be ignored.

program. This observation is the key to the Dynamic Econometric Retention Model. The original dynamic programming model is modified by the inclusion of heterogeneity and transience, and individual behavior is then optimal with respect to these factors. Now this model is statistically interesting. Given observed behavior, we wish to estimate the parameters of F and G. DERM uses a maximum likelihood procedure for obtaining these estimates, and they permit calculation of the retention rate. Notice that PEM is a misspecified version of DERM and hence must yield inferior estimates.

In summary, there are no individual differences in the dynamic programming model of Section II and hence no statistical problem. In PEM the unobserved differences among individuals are estimated, but they are not embedded in the dynamic program that determines the stay-leave decisions. DERM resolves this problem by reformulating the dynamic program to include these unobserved factors and then, through maximum likelihood, estimating the distributions of the unobserved components. With the ensuing estimates in hand, a straightforward calculation gives the retention rate.

## II. A STOCHASTIC MODEL OF RETIREMENT

The decision to stay in or leave the military depends not only on the individual's past performance but also on his anticipated future performance in a military compared with a civilian environment, in particular by the promotion probabilities from the current grade to higher grades. An officer decides to leave or stay by comparing the return from retiring with the expected return from staying one more period and then pursuing an optimal retirement policy. Leaving entails optimal sequential search in the civilian job market, the rewards from which are added to the return from the pension accumulated while the person was in the military. This description indicates that officers can solve the retirement problem by formulating it as a stochastic dynamic program.

### THE DYNAMIC RETIREMENT MODEL

Throughout the analysis, officers are assumed to be risk-neutral; that is, they choose to stay or leave solely on the basis of which choice maximizes the expected present value of future income. We made no adjustments for differences between the riskiness of military and civilian income.\* Furthermore, to concentrate on the sequential nature of the decision process, the model ignores exogenous uncertainty and unobserved differences among officers. These are considered in the estimation methods of Sections III and IV.

The dynamic retirement model has the following structure: Let  $i = 1, 2, 3, \dots, 26$  denote the 26 mutually exclusive combinations of

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\*We studied the effects of risk-aversion under the assumption that officers had constant absolute risk-aversion. For a reasonable range of riskiness in civilian incomes, the optimal retirement policy behaved as anticipated when the risk-aversion parameter was perturbed. Thus no new insights were achieved. See Gotz and McCall (1979) for details. There are, of course, several unresolved problems in the intertemporal resolution of uncertainty when the decisionmaker is risk-averse. See Dreze and Modigliani (1972).

grade, promotion timing group, and component (regular or reserve).<sup>\*</sup> In the analysis, each of these combinations is a state. The grades run from captain through colonel. For each grade above captain, each promotion timing group is a range of years of service at which promotion to that grade took place. There are four of these ranges per grade. For example,  $i = 10$  ( $i = 9$ ) represents a regular major who had been promoted to that rank in the eighth, ninth, or tenth (11th or 12th) year of service. State number 1 is reserve captain and 2 regular captain. The civilian state is numbered 27.<sup>†</sup>

Movement among the grades, promotion timing groups, and components are assumed to be generated by a first-order Markov chain with transition probabilities  $P_{ijt}$ ,  $i = 1, 2, \dots, 26$ ;  $j = 1, 2, \dots, 27$ ;  $t = 4, 5, \dots, 30$ , where  $t$  refers to year of service. Thus,  $P_{ijt}$  is the probability of going to state  $j$ , say regular major, in the next period given that the state occupied this period is  $i$ , say reserve captain, and the year of service in this period is  $t$ . Demotions are extremely rare in the Air Force, so it is assumed that  $P_{ijt} = 0$  whenever  $j < i$ . This, of course, implies that the Markov matrix  $P$  of transition probabilities is upper triangular. The upper triangular portion of the Markov matrix is also dominated by zero entries reflecting the impossibility of most one-period promotions of, for example, captain to colonel, the assumed zero probability of moving from regular to reserve component, and certain obvious restrictions on moving from one promotion timing group to another. The individual faces the Markov matrix  $P$  only if he chooses to remain at least one more year--i.e., the  $P_{ijt}$  are conditional on not voluntarily leaving the force. Note that  $P_{i,27,t}$  is the probability of being involuntarily separated or retired.

Military pay (basic pay plus basic allowances for quarters and subsistence)<sup>‡</sup> depends on grade level and year of service and is denoted

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\* Reserve component officers in this study are on active duty. They differ from regular component officers in their tenure provisions and promotion rates. Reserve officers may become regular officers.

† See Table A.1 in the appendix.

‡ Allowances are not taxable and basic pay is calculated in this study after federal income tax.

by  $m_{it}$  where the subscript ranges have been noted above. Furthermore, if an officer leaves the force from  $i$  upon completing  $t$  years of service, the fraction of basic pay that is collected per period is  $r_t$ , the pension parameter and  $0 \leq r_t < 1$ .<sup>\*</sup> At each stage of the decision process an officer in state  $i$  may leave the Air Force and receive a retirement income of  $r_t(m_{it} - a_{it})$  each period, where  $a_{it}$  is the allowances not counted in the retirement pay calculations. Search in the civilian labor market is assumed to proceed immediately, with  $W_t(i)$  denoting the optimal return from search.<sup>†</sup> In general, a different wage offer distribution,  $F_{it}$ , might be associated with each grade/year of service combination from which the individual left the Air Force, the presumption being that there is a relationship among grade achieved, age at entry into the civilian labor force, and productivity in the civilian sector. At this point we merely note that the expected discounted return from leaving the Air Force now and searching optimally in the civilian sector is given by:

$$U_t(i) \equiv r_t(m_{it} - a_{it}) \sum_{j=t+1}^{\infty} s_{tj} \beta^{j-t} + W_t(i). \quad (1)$$

The probability of surviving at least until year  $j$  given survival at  $t$  is given by  $s_{tj}$ , and  $\beta$  is the discount factor ( $\beta \equiv 1/[1 + \rho]$  where  $\rho$  is the individual's marginal rate of time preference).

If the officer chooses to remain in the Air Force, he moves according to transition probability  $P_{ijt}$  from state  $i$  to state  $j$  in the next period. If  $j \leq 26$ —i.e., he is not involuntarily separated or retired from the Air Force—then he receives the single period compensation

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<sup>\*</sup>The current formula for  $r_t$  is:

$$r_t = \begin{cases} 0 & \text{if } t < 20 \\ .025t & \text{if } 20 \leq t \leq 30 \\ .75 & \text{if } t \geq 30 \end{cases}$$

<sup>†</sup>For a discussion of this finite horizon search model, see Lippman and McCall (1976).

$m_{j,t+1}$  and again chooses whether to remain or leave and receives the optimal return of  $V_{t+1}(j)$ . The value of  $j$  is unknown, but the expected value of the single period compensation  $m_{j,t+1}$ , plus the optimal return at  $t + 1$ , is simply:

$$\sum_{j=1}^{26} s_{t,t+1} P_{ijt} [m_{j,t+1} + V_{t+1}(j)] . \quad (2)$$

To obtain the value of this return at period  $t$ , it is discounted by  $\beta$ ; thus, the total return from staying in and behaving optimally for the remaining period (if  $P_{i,27,t} = 0$ ) is

$$\beta \sum_{j=1}^{26} s_{t,t+1} P_{ijt} [m_{j,t+1} + V_{t+1}(j)] . \quad (3)$$

If there is a nonzero probability that the officer will be terminated even if he desires to remain, then the return associated with becoming a civilian must be added to (3):

$$\beta \sum_{j=1}^{26} s_{t,t+1} P_{ijt} [m_{j,t+1} + V_{t+1}(j)] + P_{i,27,t} [\beta s_{t,t+1} x_{it} + U_t(i)] \quad (4)$$

where  $x_{it}$  is any severance pay associated with the involuntary separation.\* Expression (4) is the return from choosing to remain in the Air Force at least one more year and behaving optimally for the remaining periods.

The optimal decision at  $t$ , stay or leave, is obtained by choosing the maximum of (1) and (4). Thus, we have derived the following functional equation:

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\* In the current system, severance pay  $x_{it}$  is paid only to those not eligible to retire, so if  $r_t$  is positive,  $x_{it}$  is zero.

$$V_t(i) = \max \left\{ \beta \sum_{j=1}^{26} s_{t,t+1} P_{ijt} [m_{j,t+1} + V_{t+1}(j)] \right. \\ \left. + P_{i,27,t} [\beta s_{t,t+1} x_{it} + U_t(i)] ; U_t(i) \right\} \quad (5a)$$

where  $V_t(i)$  is the expected discounted return when the decisionmaker (officer) is in state (grade level)  $i$  and follows an optimal retirement strategy.

For each state ( $i \leq 26$ ) there is a mandatory separation or retirement year of service  $T_i$ —i.e.,  $P_{i,27,T_i} = 1.0$ . Hence,

$$V_{T_i}(i) = U_{T_i}(i) \quad i = 1, \dots, 26 \quad (5b)$$

It was first thought that the optimal retirement policy would have a fairly simple structure. So far, this has not proved to be the case.\* For this reason it was decided to perform a numerical analysis of a modified version of (5). Search has been eliminated from the functional equation by replacing  $W_t(i)$  with

$$\sum_{j=t+1}^T s_{tj} \beta^{j-t} w_{ij}$$

where  $w_{ij}$  are the civilian wages received when the officer has achieved rank  $i$  at retirement and the time since retirement is  $j - t + 1$ .  $T$  is taken to be the year of service equivalent of 65 years old. In addition to the elimination of search, expression (5) assumes that officers have perfect information about promotion, augmentation,<sup>†</sup> and force-out/mandatory retirement probabilities and civilian wages.<sup>‡</sup>

\* Given the current retirement pay structure, certain propositions can be developed for those who are not yet eligible to retire. See Gotz (forthcoming). However, the usefulness of the current study is in predicting retention and retirement behavior under alternative retirement systems.

† Augmentation is the movement from reserve to regular component.

‡ The assumption of perfect information about  $P$ , the transition matrix, is not very stringent. The *Air Force Times* publishes detailed



Next we consider a numerical analysis using the following functional equation:

$$\begin{aligned}
 V_t(i) = \max \left\{ \beta \sum_{j=1}^{26} s_{t,t+1} P_{ijt} [m_{j,t+1} + V_{t+1}(j)] \right. \\
 + P_{i,27,t} \left[ \beta s_{t,t+1} x_{it} + r_t (m_{it} - a_{it}) \sum_{k=t+1}^{\infty} s_{tk} \beta^{k-t} \right. \\
 + \sum_{k=t+1}^T s_{tk} \beta^{k-t} w_{ik} \left. \right]; r_t (m_{it} - a_{it}) \sum_{k=t+1}^{\infty} s_{tk} \beta^{k-t} \\
 \left. + \sum_{k=t+1}^T s_{tk} \beta^{k-t} w_{ik} \right\}. \quad (6)
 \end{aligned}$$

#### NUMERICAL RESULTS

The numerical analysis of the functional equation (6) is unique in that it contains data on the promotion, augmentation, and force-out/mandatory retirement probabilities,  $P_{ijt}$ , from the Air Force's Uniform Officer Records, plus data on military compensation,  $m_{it}$ ,\* and the pension parameters,  $r_t$ . Data on civilian wages,  $w_{it}$ , were obtained from Rand's Medical Survey of Retired Military Personnel and the Bureau of the Census Current Population Survey for professional, technical, and kindred workers excluding obvious noncorresponding occupations (e.g., medical doctors, dentists). Unless stated otherwise, the discount rate,  $\rho$ , is set at .10.

At each stage (year of service) of the process the officer evaluates (6) and either stays in the Air Force for at least one more year or leaves basing his decision on which choice maximizes the expected present value of future income. In effect, we are calculating the present breakdowns of promotions by component, aeronautical rating, etc. Also, the infrequent changes in promotion policies are usually known in advance.

\* Military pay schedules are published periodically in the *Air Force Times* and other military-oriented publications.

value and decision for the officer who maximizes expected present value and is facing the mean civilian wage path for retired military personnel. Needless to say, not all officers display this "representative" behavior.

We have examined a wide range of combinations of rating, source of commission, and fiscal year. However, for ease of presentation we concentrate on the base case, which considers the optimal behavior of the "representative" nonrated officer who accessed through ROTC or OTS/OCS. The other combinations examined do not differ in any fundamental way from this case.

The retirement plan has the following features: If the officer voluntarily leaves before completing 20 years of service, he receives no retirement benefits; if he retires on completion of 20 years, he receives 50 percent of the basic pay ( $m_{i,20} - a_{i,20}$ ) associated with his highest grade; for every year after 20 the pension parameter is augmented by 2-1/2 percentage points up to a maximum of 75 percent at 30 years of service. The Markov matrix,  $P$ , is based on empirical promotion, augmentation, and force-out/mandatory retirement rates from fiscal year 1970. The military pay scales are also for fiscal year 1970, and civilian pay has been adjusted to correspond to the same year.

The numerical results from the base case are presented in Table 1. Rather than presenting all promotion groups and components, we present only regular component "due course" officers--i.e., those promoted to their current grades in the modal year of service.

The first column of the table shows completed years of service. We emphasize the retirement behavior of majors, lieutenant colonels, and colonels, but for reference we include the optimal decision for captains after seven years of service (stay); the discounted expected return of following an optimal policy, \$142,000--i.e., staying for one more year and following an optimal retirement strategy thereafter; and the cost of making an incorrect decision, \$34,000, which here would be leaving the Air Force after seven years of service. The three entries in each year of service for majors have a corresponding interpretation. Calculations of the cost of making an incorrect decision assume that the individual does behave optimally after the mistake. This has no

Table 1

OPTIMAL RETURNS AND COSTS OF LEAVING, NONRATED REGULAR OFFICERS  
(Thousands of 1971 dollars)

Completed Years of Service	Captain	Major	Completed Years of Service	Lieutenant Colonel	Colonel
7	Stay 142 34		20	Stay 175 7	
12		Stay 155 45	21	Stay 176 6	
13		Stay 157 48	22	Stay 175 2	S/L 199 0
14		Stay 160 51	23	S/L 175 0	Stay 201 1
15		Stay 163 54	24	S/L 175 0	Stay 203 3
16		Stay 165 57	25	S/L 176 0	Stay 206 4
17		Stay 157 49	26	Leave 177 -1	Leave 209 -2
18		Stay 157 50	27	Leave 177 -2	Leave 209 -2
19		Stay 158 52	28	Mand. Retire. 177	Leave 210 -2
20		Leave 160 -1	29		Leave 209 -3
21		Leave 161 -1	30		Mand. Retire. 209
22		Mand. Retire. 162			

effect on the calculation for those who incorrectly leave the Air Force several years before the optimal point, but it does affect the calculations for those who incorrectly stay.\*

To facilitate understanding, we have signed the cost of making an incorrect decision by calculating it as the return associated with remaining in the Air Force for at least one year *minus* the return associated with leaving. The signed cost may then be interpreted as the cost of leaving the military even though that cost may be negative. The cost of leaving,  $\bar{c}_t(i)$ , is calculated as follows:

$$\begin{aligned} \bar{c}_t(i) = & \beta \sum_{j=1}^{26} s_{t,t+1} P_{ijt} [m_{j,t+1} + v_{t+1}(j)] \\ & + P_{i,27,t} [\beta s_{t,t+1} x_{it} + U_t(i)] - U_t(i). \end{aligned} \quad (7)$$

This cost plays a crucial role in the estimation procedures of Sections III and IV. Clearly, when this cost is positive (negative) the return from staying is greater (less) than the return from leaving.

The common conception that retirement pay is an overwhelming inducement for officers beyond the tenth year of service to remain in the force appears to be correct. The optimal retention policy for majors--optimal in the sense of maximizing expected present value--(reserve and regular) is to stay until they complete 20 years of service and then retire. For a regular major with 19 years of service, the discounted expected return of following an optimal policy is \$158,000 and the difference between staying and leaving is \$52,000. After an individual is eligible for a 50 percent pension at 20 years of service, the difference between leaving (the optimal decision) and staying is

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\*The pattern of optimal decisions need not obey a myopic control limit rule of the form "retire if  $x \geq \xi$  and stay otherwise." This is because military pay  $m_{it}$  is not strictly increasing with  $t$ . Therefore, if the individual mistakenly remains in the service, it may be optimal to remain even longer in order to capture the effect of the next pay raise before retiring. The conditions for the optimality of myopic control limit rules are stated in Lippman and McCall (1976).

quite small, roughly \$1,000 after 20 and 21 years of service. We believe that the sizes of the retention rate and the cost of leaving the Air Force are related; therefore our calculations indicate that although we should never observe a major quitting after 19 years of service, we may very well see some desiring to stay in beyond 22, the small advantage to leaving being offset by factors not measured with our data. In fact, voluntary retention rates among majors rarely fall below 97 percent in the "teen" years of service.

The optimal retirement policy for lieutenant colonels is for regular officers to stay at least until completing their 23d year of service and for reserve officers to stay until completing their 22d year of service. The difference between the optimal policies is that regulars have a higher probability of being promoted to colonel than reserves. For a regular due-course lieutenant colonel with 22 years of service, the discounted expected return of following an optimal policy is \$175,000 and the difference between staying and leaving is \$2,000. From 22 until 27 years of service, the cost of making the wrong decision for regulars varies from less than \$500 to \$2,000. For most cases, the loss is less than \$1,000. Other factors not measured by our data could cause lieutenant colonels in this age interval to make the "wrong" decision.

The optimal retirement policy for colonels (regular and reserve) is to stay until they complete 26 years of service. For a colonel with 25 years of service, the discounted expected return from following an optimal policy is \$206,000 and the difference between staying and leaving is \$4,000. The cost of remaining in the Air Force from 26 to 29 years of service ranges between \$2,000 and \$3,000.

The differences in the optimal decisions between reserve and regular lieutenant colonels and between lieutenant colonels and colonels are important in that they illustrate the effect of pay patterns on behavior. The reserve lieutenant colonel with no chance of being promoted to colonel is induced to remain until completing 22 years by the longevity pay increase at 22 years. By the same token, the colonel faces his last longevity pay increase at 26 years and the "representative" colonel is induced to remain at least that long. For the regular

lieutenant colonel, the chance of being promoted to colonel involves the chance of both higher active duty pay and higher retirement pay, thereby inducing the officer to remain in the Air Force.

The costs of making the "wrong" decision for these officers are small compared with the optimal returns, which are generally greater than \$150,000. Therefore, one cannot expect a pattern of retirements wherein all officers in a given grade and component retire in the same year of service. Nevertheless, for those retiring in fiscal year 1970, both the median and mean completed years of service at the time of retirement for regular colonels (nonrated, nonacademy) were between 26 and 27. For lieutenant colonels the median completed year of service was between 23 and 24 and the mean was between 24 and 25. Thus Air Force retention statistics do behave as if the average officer was making his retirement decision in an optimal sequential fashion. This gives us confidence in the model and also in any predictions we might make about changes in the retirement parameters.

We also studied variations in the optimal retirement policy induced by changes in civilian pay, military pay, and the discount rate. The optimal policy was sensitive only to extremely large changes in civilian or military compensation rates. One reason for these robust results is the assumption that individuals making mistakes in the current period will behave optimally in subsequent periods.\*

The empirical analysis just presented assumes that individuals are identical. In the next section, we consider the complications that occur when differences are explicitly acknowledged.

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\*Using this same model, we also examined the effects of alternative retirement systems on the incentives to retire. In particular, we evaluated the Retirement Modernization Act and the proposal of the President's Commission on Military Compensation. The interested reader should consult Gotz and McCall (1979).

### III. A PRELIMINARY ECONOMETRIC METHOD FOR ESTIMATING THE RETENTION RATE

The dynamic model of retirement assumed that individuals are identical and not affected by any exogenous uncertainty. In fact, characteristics that influence retirement will differ across individuals and each person will be affected by transient variables that are beyond his control. Thus, aggregate retirement behavior is influenced by two hitherto neglected factors. Population heterogeneity causes individuals to respond differently to identical environmental changes, whereas transient variables themselves generate differential behavior. In the sequel, it is assumed that the heterogeneous factors are constant over time for each individual. Each individual knows the value of his own heterogeneous factor. However, the transient effect is a random variable with a known probability distribution. If we are to develop the correct statistical method, the estimation procedure must explicitly consider heterogeneity and transience.

In estimating the retention rate, we present two different econometric methods for analyzing these heterogeneous and random effects. Although somewhat redundant, this presentation reflects the actual development of the estimation procedure, and we hope it thereby enhances the exposition. The Preliminary Econometric Method presented in this section explicitly considers the statistical effect of the heterogeneous and transient factors and obtains maximum likelihood estimators of the retirement parameters. The Dynamic Econometric Retention Model yields an estimation method that differs fundamentally from the PEM in that individual retirement behavior is assumed to be optimal with respect to both the known heterogeneous factor and the distribution of the transient random variable. DERM is the subject matter of Section IV.

We now present a method<sup>\*</sup> for estimating the retirement rate when the statistical assumptions do not correspond with those of the dynamic

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<sup>\*</sup>The methods presented here and in Section IV are based on the important work of Heckman and Willis (1976).

economic model. That is, methods are devised for obtaining maximum likelihood estimators of the retirement parameters when the statistical model contains both heterogeneity and transient factors. However, neither of these factors enters into the calculation of the individual's optimal retirement policy. The modeling of individual behavior and the econometric modeling are two distinct enterprises.

By exploiting the longitudinal data, this econometric method is able to estimate the heterogeneity distribution based on differences that are specific to an individual and invariant over time. In the present application, this heterogeneity includes the elusive "taste for the military" that has been so extremely difficult to measure with cross-sectional data.

Let  $c_{mt}(i)$  be the cost of leaving the Air Force for the  $m$ th individual in state  $i$  with  $t$  years of service. We assume that  $c_{mt}$  can be decomposed in the following way:\*

$$c_{mt}(i) = \bar{c}_t(i) + \delta_m + \epsilon_{mt},$$

$$c_{mt}(i) = \bar{c}_t(i) + \alpha + \hat{\delta}_m + \hat{\epsilon}_{mt},$$

where  $\bar{c}_t(i)^\dagger$  is the average cost of leaving calculated for individuals with  $t$  years of service who occupy state  $i$ ,  $\alpha$  is the sum of the expectations of  $\delta$  and  $\epsilon$ ,  $\delta_m$  is the fixed effect for the  $m$ th individual, and  $\epsilon_{mt}$  is the transitory disturbance. Both  $\delta_m$  and  $\epsilon_{mt}$  are random variables with means and variances given by

$$E(\delta_m) = \alpha_1 \quad E(\epsilon_{mt}) = \alpha_2$$

$$\text{Var}(\delta_m) = \sigma_\delta^2 \quad \text{Var}(\epsilon_{mt}) = \sigma_\epsilon^2$$

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\*This is a random effects model in that the persistent unobservables  $\delta_m$  are independent of  $\bar{c}_t(i)$ , the cost of leaving.

<sup>†</sup>This is the same cost of leaving that was derived above in Eq. (7).



with  $\delta_m \equiv \hat{\delta}_m + \alpha_1$  and  $\epsilon_{mt} \equiv \hat{\epsilon}_{mt} + \alpha_2$ . The covariance among the random variables is assumed to satisfy:

$$E(\hat{\epsilon}_{mt} \hat{\epsilon}_{mt'}) = \begin{cases} 0 & t \neq t' \\ \sigma_{\epsilon}^2 & t = t', \end{cases}$$

$$E(\hat{\delta}_m \hat{\epsilon}_{nt}) = 0, \text{ for all } m, n,$$

and

$$E(\hat{\delta}_m \hat{\delta}_n) = \begin{cases} 0 & m \neq n \\ \sigma_{\delta}^2 & m = n. \end{cases}$$

It follows that the random variable

$$\eta_t = \hat{\delta}_m + \hat{\epsilon}_{mt}$$

has the following stochastic structure

$$E(\eta_t) = 0$$

and

$$E(\eta_t \eta_s) = \begin{cases} \sigma_{\delta}^2 & t \neq s \\ \sigma_{\delta}^2 + \sigma_{\epsilon}^2 & t = s. \end{cases}$$

Letting the correlation between  $\eta_t$  and  $\eta_{t+1}$  be given by  $\rho$ , we see that

$$\rho = \frac{\sigma_{\delta}^2}{\sigma_{\delta}^2 + \sigma_{\epsilon}^2}.$$

Thus, if the persistent factor, the source of the heterogeneity, is of little consequence, this correlation coefficient is small. However, if the persistent factor is important but ignored, serious omitted variables biases can result in statistical modeling of stay or leave decisions.\*

In the retirement model, it is assumed that whenever the individual's true cost of leaving exceeds zero he stays, and whenever it falls below zero he retires.

Letting  $S_t(i)$  denote stay at  $t$  when state  $i$  is occupied and  $L_t(i)$  leave, the event  $E$ , stay till time  $t$  and then leave, is denoted by:

$$E \equiv \{S_1(i), \dots, S_{t-1}(j), L_t(j)\}.$$

State  $j$  is the military state occupied at the time of retirement.<sup>†</sup> Each unique sequence of states occupied is an event.

It is possible to calculate the probability of  $E$  by obtaining the conditional probability  $P\{E|\hat{\delta}_m\}$  and then integrating over the distribution of  $\hat{\delta}$ . Now

$$\begin{aligned} P\{E|\hat{\delta}\} &= P\{c_{m,1}(i) > 0, \dots, c_{m,t-1}(j) > 0, c_{mt}(j) < 0|\hat{\delta}_m\} \\ &= P\{c_{m1}(i) > 0|\hat{\delta}_m\} \dots P\{c_{m,t-1}(j) > 0|\hat{\delta}_m\} P\{c_{mt}(j) < 0|\hat{\delta}_m\}, \end{aligned} \quad (8)$$

so

$$P\{E\} = \int_{-\infty}^{\infty} P\{E|\hat{\delta}_m\} dH(\hat{\delta}_m), \quad (9)$$

where  $H$  is the cumulative distribution function of  $\hat{\delta}_m$ .

\*For a discussion of this, see Heckman (1978a, b).

<sup>†</sup>If the retirement is mandatory at, say, years of service  $t'$ , then the event "stay until mandatory retirement" is  $\{S_1(i), \dots, S_{t'-1}(j)\}$ .

If there are  $K$  events in a sample and  $n_k$  individuals are observed for the  $k$ th event, then the sample likelihood is

$$L = \prod_{k=1}^K [P(E_k|\theta)]^{n_k}, \quad (10)$$

where  $\theta$  is the parameter vector to be estimated. If  $\hat{\delta}$  and  $\hat{\epsilon}$  are both assumed to be normal, then the maximum likelihood estimators of  $\alpha$ ,  $\sigma_{\hat{\delta}}^2$ , and  $\sigma_{\hat{\epsilon}}^2$  are obtained by maximizing this expression with respect to  $\alpha/(\sigma_{\hat{\delta}}^2 + \sigma_{\hat{\epsilon}}^2)^{1/2}$ ,  $(\sigma_{\hat{\delta}}^2 + \sigma_{\hat{\epsilon}}^2)^{-1/2}$ , and  $\rho$ .<sup>\*</sup> Having obtained estimates of these distribution parameters, we can then estimate the retention rate.

In this statistical formulation, retention rates by year of service depend on both the future and the past. They depend on the future through the dynamic program;  $\bar{c}_t(i)$  is a present value. They depend on the past because the mean value of  $\hat{\delta}$  for those remaining in the force is lower if past retention rates were high and higher if they were low. It is possible to calculate the retention rate by computing the probability of remaining at least  $t$  years and then conditioning that probability on remaining at least  $t-1$  years. For example, the retention rate at the  $t$ th year of service, ignoring the state dimension, is:

$$RET_t \equiv P\{S_t | S_1, \dots, S_{t-1}\} = \frac{\int_{-\infty}^{\infty} \left[ \int_{-\bar{c}_1 - \hat{\delta} - \alpha}^{\infty} dF(\epsilon) \dots \int_{-c_t - \hat{\delta} - \alpha}^{\infty} dF(\epsilon) \right] dH(\hat{\delta})}{\int_{-\infty}^{\infty} \left[ \int_{-\bar{c}_1 - \hat{\delta} - \alpha}^{\infty} dF(\epsilon) \dots \int_{-\bar{c}_{t-1} - \hat{\delta} - \alpha}^{\infty} dF(\epsilon) \right] dH(\hat{\delta})}, \quad (11)$$

where  $F$  is the distribution function of  $\epsilon$ . Clearly,  $RET_t$  will be independent of previous retention rates only if  $H$  has unit mass at a point--if individuals have no differences in tastes for the service.

<sup>\*</sup> See Heckman and Willis (1976).

#### IV. THE DYNAMIC ECONOMETRIC RETENTION MODEL

The Dynamic Econometric Retention Model (DERM) unifies the stochastic dynamic retirement model (Section II) and the Preliminary Econometric Method (Section III). DERM is a sequential model of economic behavior that contains the appropriate econometric method for estimating the retention rate. Our procedure is novel in that the econometric method is endogenously determined by the stochastic specification of the economic model. In DERM the individual is assumed to know his preference for the service and the distribution of the transient factor. The econometric method described above (PEM) ignores this knowledge.

In reformulating the dynamic program presented in Section II, we can more easily convey the main ideas if we emphasize a single grade, thereby ignoring the promotion probabilities. We also ignore the survival probabilities. In keeping with the previous section, we consider two independently distributed random variables,  $\gamma$  and  $\epsilon$ . The random variable  $\gamma$  can be thought of as denoting the monetary equivalent of the annual nonmonetary returns associated with being in the military (net of nonmonetary returns accruing to civilians). Each individual has a known (to him) value of  $\gamma$  that remains constant over time. The presence of  $\gamma$  means individual differences in preference for service life will be revealed by differences in optimal plans. For example, a change in the value of a pay raise at 27 years of service is more likely to change the retention pattern of someone who planned to remain until 26 years than of someone who planned to leave at 20 years. The random variable  $\epsilon$  is the monetary equivalent of a transient disturbance. These transient disturbances will in general cause a divergence between the previous retirement plan, which ignored  $\epsilon$ , and one that explicitly considers  $\epsilon$ . A subscripted value of  $\epsilon$ --e.g.,  $\epsilon_{it}$ --indicates an actual value of  $\epsilon$  known to individual  $i$  at time  $t$ .

Let  $V_t(\gamma_m, \epsilon_{mt})$  be the optimal return for the individual with  $\gamma = \gamma_m$  who has just observed the transient disturbance  $\epsilon_{mt}$ . The optimal return is the solution to the following functional equation,

$$V_t(\gamma_m, \epsilon_{mt}) = \text{MAX} \left\{ \epsilon_{mt} + \beta \gamma_m + \beta m_{t+1} + \beta E_\epsilon [V_{t+1}(\gamma_m, \epsilon)]; U_t \right\}, \quad (12)$$

where  $m_{t+1}$  is military pay for year of service  $t + 1$ ,  $\beta$  is the discount factor,  $U_t$  is the return from leaving the service, and  $E_\epsilon$  denotes the expectation with respect to the distribution of the transient disturbance. Including the Markov chain in the formulation yields the following functional equation:

$$V_t(i, \gamma_m, \epsilon_{mt}) = \text{MAX} \left\{ \epsilon_{mt} + \beta \sum_{j=1}^{26} P_{ijt} \{ \gamma_m + m_{j,t+1} + E_\epsilon [V_{t+1}(j, \gamma_m, \epsilon)] \right. \\ \left. + P_{i,27,t} (\beta x_{it} + U_t(i)); U_t(i) \right\}. \quad (13)$$

The expectation of the optimal return at  $t + 1$  is taken because the individual cannot know in advance what values future disturbances will take. This expectation for Eq. (12) is given by

$$E_\epsilon [V_{t+1}(\gamma_m, \epsilon)] = \int_{-\infty}^{\infty} V_{t+1}(\gamma_m, \epsilon) dF(\epsilon) \\ = \int_{-c_{t+1}(\gamma_m)}^{\infty} \left( \epsilon + \beta \gamma_m + \beta m_{t+2} + \beta E_\epsilon [V_{t+2}(\gamma_m, \epsilon)] \right) dF(\epsilon) \\ + U_{t+1} \int_{-\infty}^{-c_{t+1}(\gamma_m)} dF(\epsilon), \quad (14)$$

where  $F(\epsilon)$  is the distribution function for  $\epsilon$ . The expected cost of leaving at  $t + 1$  for individual  $m$  in his  $t$ th year under the assumption that  $\epsilon$  has mean zero is denoted by  $c_{t+1}(\gamma_m)$  and is defined by

$$c_{t+1}(\gamma_m) \equiv \beta \gamma_m + \beta m_{t+2} + \beta E_{\epsilon} [V_{t+2}(\gamma_m, \epsilon)] - U_{t+1}. \quad (15)$$

The integral from  $-\infty$  to  $-c_{t+1}(\gamma_m)$  of  $dF(\epsilon)$  is the individual's estimate of the probability of leaving the service at  $t + 1$ , given that today is  $t$ . This loss probability is a function of his taste for the service,  $\gamma_m$ , as well as a function of the distribution of transient disturbances. Therefore, there is no unique future retention rate that can be used to construct a present value cost-of-leaving number for a cohort.

Proceeding in the same manner as in the preceding section, denote stay at  $t$  by  $S_t(i)$  and leave by  $L_t(i)$  when state  $i$  is occupied. The event  $E$  is given by

$$E \equiv \{S_1(i), \dots, S_{t-1}(j), L_t(j)\}.$$

State  $j$  is the military state occupied at the time of retirement. Each unique sequence of states occupied and decisions made is an event. The probability of an event,  $P\{E\}$ , is given by

$$P\{E\} = \int_{-\infty}^{\infty} P\{E|\gamma_m\} dG(\gamma_m), \quad (16)$$

where  $G(\cdot)$  is the distribution function for  $\gamma$ .

If  $E = \{S_1(i), S_2(i)\}$ , the probability of this event may be written as

$$P\{S_1(i), S_2(i)\} = \int_{-\infty}^{\infty} \left[ \int_{-c_1(i, \gamma_m)}^{\infty} dF(\epsilon) \int_{-c_2(i, \gamma_m)}^{\infty} dF(\epsilon) \right] dG(\gamma). \quad (17)$$

If there are  $K$  events in a sample and  $n_k$  individuals are observed for the  $k$ th event, the sample likelihood function is

$$L = \prod_{k=1}^K \left[ P\{E_k | \omega\} \right]^{n_k}, \quad (18)$$

where  $\omega$  is the parameter vector to be estimated. If  $F$  and  $G$  are both normal cumulative distribution functions, then the elements of  $\omega$  are  $\mu_\epsilon + \mu_\gamma$ ,  $\sigma_\epsilon^2$ , and  $\sigma_\gamma^2$ . (The means of  $\epsilon$  and  $\gamma$  are not separately identifiable.)

Note that the likelihood value depends on the costs of leaving,  $c_t(i, \gamma)$ , but that  $c_t(i, \gamma)$  in turn depends on the parameters of  $F(\cdot)$ , the distribution function for  $\epsilon$ . If we ignore the state dimension, expansion of (15) yields:

$$c_t(\gamma_m) = \beta \gamma_m + \beta m_{t+1} + \beta \left[ \int_{-c_{t+1}(\gamma_m)}^{\infty} \left( \epsilon + \beta \gamma_m + \beta m_{t+2} + \beta E_\epsilon [V_{t+2}(\gamma_m, \epsilon)] \right) dF(\epsilon) + U_{t+1} \int_{-\infty}^{-c_{t+1}(\gamma_m)} dF(\epsilon) \right] - U_t. \quad (15a)$$

If  $\epsilon$  is normal with mean zero and variance  $\sigma_\epsilon^2$ , then  $c_t(i, \gamma_m)$  depends on  $\sigma_\epsilon^2$ .

Because  $c_t(i, \gamma)$  is not a simple function of the parameters of  $F$ , the estimation technique is the following. For each value of  $\sigma_\epsilon^2$ , the stochastic dynamic program is solved for  $c_t(i, \gamma)$  for all  $i$ ,  $t$ , and for selected values of  $\gamma$ . The likelihood function is then maximized (conditional on  $\sigma_\epsilon^2$ ) to obtain estimates of  $\mu_\epsilon + \mu_\gamma$  and  $\sigma_\gamma^2$ . The maximum likelihood estimates are those values of  $\sigma_\epsilon^2$ ,  $\mu_\epsilon + \mu_\gamma$ , and  $\sigma_\gamma^2$  yielding the highest value of the conditional likelihoods.

As in the preceding section, retention rates in this model depend on both the future and the past. The expected retention rate at the  $t$ th year of service, ignoring the state dimension, is given by

$$RET_t = P\{S_t | S_1, \dots, S_{t-1}\} =$$

$$\frac{\int_{-\infty}^{\infty} \left[ \int_{-c_1(\gamma_m)}^{\infty} dF(\epsilon) \cdot \dots \cdot \int_{-c_{t-1}(\gamma_m)}^{\infty} dF(\epsilon) \int_{-c_t(\gamma_m)}^{\infty} dF(\epsilon) \right] dG(\gamma_m)}{\int_{-\infty}^{\infty} \left[ \int_{-c_1(\gamma_m)}^{\infty} dF(\epsilon) \cdot \dots \cdot \int_{-c_{t-1}(\gamma_m)}^{\infty} dF(\epsilon) \right] dG(\gamma_m)} . \quad (19)$$

$RET_t$  will be independent of  $RET_{t-1}, RET_{t-2}, \dots$ , only if  $G(\cdot)$  has unit mass at a point--if there are no permanent differences in tastes among individuals.



## V. COMPARISON OF ESTIMATION PROCEDURES

### DIFFERENCES BETWEEN PEM AND DERM

Although the similarities between PEM and DERM are striking, the differences are fundamental. They reside in the treatment of the persistent and transient factors affecting individual decisionmaking. If these factors exist, then behavior should, by hypothesis, be optimal with respect to them.

For each method, consider the size of the military pay change required to move an individual from indifference between staying and leaving. Suppose that the individual is in year of service  $t$  and the military pay change will occur in year of service  $t + 2$ . For PEM, indifference between staying and leaving means  $\bar{c}_t + \delta + \varepsilon = 0$ . If  $\bar{c}_{t+1}$  is greater than zero, then any small increase in military pay will increase the optimal return at  $t + 1$ ,  $V_{t+1}$  and therefore increase  $\bar{c}_t$ , which moves the individual from indifference. However, if  $\bar{c}_{t+1}$  is negative, then the value of  $V_{t+1}$  can be increased only if the size of the pay increase exceeds  $-\bar{c}_{t+1}$ . This asymmetry of the effect of a small pay change does not occur in DERM. For DERM, indifference between staying and leaving means  $c_t(\gamma) + \varepsilon = 0$ . A small change in military pay in year of service  $t + 2$  increases  $E_\varepsilon[V_{t+1}(\gamma, \varepsilon)]^*$  regardless of the value of  $c_{t+1}(\gamma)$  and therefore increases  $c_t(\gamma)$ , which moves the individual from indifference.

There are two sources of the asymmetry above. First, transient disturbances come as complete surprises to the individual in PEM. Since he does not conceive of disturbances as altering planned decisions, he views his future state-contingent exit years as known with certainty. He does not account for the probability that he may remain in the military longer or shorter than expected. Second, although the individual is aware of his own taste for the military,  $\delta$ , in the current period, he does not account for that taste in planning his future behavior. Therefore, except for those whose values of  $\delta$  equal zero, individuals will systematically not follow their own plans in PEM.

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\* See expression (13).

AN APPLICATION OF DERM TO A POLICY CHANGE WITH GRANDFATHERING

Not uncommon to proposed retirement system alternatives is some form of "grandfathering" members of the current force when the new system is begun. For example, they might have the option of serving out their careers under the current system rules, leaving under the current system rules, or serving some additional time to become eligible to remain or leave under the new system rules. Although a regression approach and PEM cannot predict the proportions choosing each alternative, they may be directly estimated using DERM. The expected future retention patterns generally will differ under different retirement plans; it is therefore of interest to know these proportions before the retirement plan and grandfathering provisions have been adopted.

Retention rates among those with the option to choose between systems should be initially higher than they would be under either the old system or the new system without grandfathering, because of the expansion of opportunities faced by these individuals. There may be those who prefer the new system to leaving and prefer leaving to the old system. There may also be those who prefer the old system to leaving and prefer leaving to the new system. One of these two groups would be lost if there were not two alternatives from which to choose. Under the President's Commission on Military Compensation (PCMC) plan, those with low values of  $\gamma$  might choose the PCMC retirement system because it increases near term benefits (10 through 19 years of service) at the expense of intermediate term benefits (20 through 29). Those with very high values of  $\gamma$  might also choose the PCMC system because the returns to remaining through 30 years of service are slightly higher than under the current system (at least for officers allowed to remain until 30). However, those with values of  $\gamma$  between the two extremes might prefer the current system because of the much larger returns than under the PCMC system to leaving after 20 but fewer than 30 years of service.

Estimating the proportion preferring each alternative is simply a matter of evaluating the stochastic dynamic program presented in DERM in which there would be three choices (current system, PCMC system, leave) rather than two. The ranges of  $\gamma$  for which each alternative is preferred can be determined from the dynamic program. If the

distribution functions for the random variables  $\gamma$  and  $\epsilon$  are known, it is possible to calculate the probability associated with each range.

#### APPRAISAL OF SOME RECENT REGRESSION STUDIES

One of the more important properties of DERM is that individuals persistently differ from one another. Consider the implication of having a frequency distribution of tastes across individuals. Even if the cost of leaving for each value of  $\gamma$  remained unchanged over some range of years of service, retention rates would rise with years of service in this range. It can be shown that the average value of  $\gamma$  for those remaining in the force increases at a decreasing rate with years of service. Larger negative values of the transient disturbance  $\epsilon$ , hence less probable values, are required to induce individuals to leave the force as years of service increase. Even without changed incentives to remain in the force, retention rates will rise with years of service.

Now suppose that for each value of  $\gamma$  the cost of leaving increases at a constant or increasing rate with years of service. This is generally the case for the years of service before retirement eligibility. If one ran a regression of retention rates against  $\bar{c}_t$ , the cost of leaving from the simple dynamic program presented in Section II, the estimated slope of the relationship would be biased upward. The change in the cost of leaving from year of service to year of service is positively, but imperfectly, correlated with the increasing mean of the taste distribution. As a consequence, there is an omitted variable bias in the regression, and the coefficient of the cost of leaving is biased upward. Further, because the cost of leaving is imperfectly correlated with the mean of the taste distribution, the regression coefficient picks up only the effect of the average change in the mean of the taste distribution. Because the mean of the taste distribution increases more rapidly in the earliest years of service than in later years, a regression model should overpredict early year of service retention rates and underpredict later ones.

Chipman and Mumm (1978) and Warner (1978) have estimated such regressions using cross-sections of retention rates and costs of leaving by year of service for enlisted military personnel. Both studies

included year of service as an additional explanatory variable, although Warner estimated the regression without year of service as well. The elasticity of the retention rate at the initial decision point (the first time individuals are eligible to leave voluntarily) with respect to military pay ranged from 7 in Warner's study to 15 in Chipman and Mumm's. There are much larger elasticities than have been found in the past. Enns (1977) and Wilburn (1970), and others found elasticities ranging between 1 and 3 at the initial decision points. These latter studies used cross-sections and time series of retention rates *only* at the initial decision point, so they do not suffer from the biases discussed above, although they are not based on any theory of optimal sequential decisionmaking and may therefore be subject to other problems.

Including year of service in the regression was found to reduce the estimated coefficient of the cost of leaving. Year of service is positively correlated with the cost of leaving and with the mean of the taste distribution. However, since year of service increases at a constant rate, the problem of overprediction of early retention rates and underpredictions of later rates should remain. Private communications with Chipman and Warner disclosed that they both did indeed have this problem in their estimates.

Our conclusions regarding overestimation of the elasticity of retention with respect to pay will not necessarily hold if different measures of the returns to staying and leaving are used in regressions. Conclusions with respect to the under- and overestimation of retention rates still obtain, however. Warner, for example, also estimated a regression of retention rates on an alternative measure he termed the average cost of leaving. Although the estimate of the elasticity at the initial decision point was lower than when  $\bar{c}_t$  was used, the regression still overpredicted early and underpredicted later service retention rates.

Retention rates depend not only on prospective future returns to remaining in the military but also to occurrences in the past. The regression models ignore the past, and retention rates are treated as empirical first-order Markov transition probabilities. This point

leads to the assessment of regression model predictions of retention rates under alternative military retirement systems.

The *Report of the President's Commission on Military Compensation* (1978) recommended changing the current retirement system, including reducing the value of benefits received upon the completion of 20 through 29 years of service and vesting progressively increasing proportions of the retirement benefits after 10 years of service.

Our model implies that, even without a reduction in retirement annuities for completing 20 or more years of service, the percentage of those completing 10 years who also complete 20 years will decline relative to the current percentage. The regression models would predict no change or an increase in this percentage. The reason our model predicts a decline is that people who were induced to remain in the service *only* because of the early vesting provision have, on the average, lower tastes for the military. After having become eligible to withdraw their vested retirement benefits, they will leave at higher rates than those who would have remained until at least 10 years even without earlier vesting. The regression models do predict a decline in retention rates, but only because of the reduction in 20 or later years of service retirement annuities. Thus, they underestimate the decline in retention.

The regression models, then, overpredict the expected years of service per new entrant and thereby underpredict the number of new entrants required to maintain a force of the same capability and, what is the same thing, overpredict the number of persons in the later years of service under the PCMC proposal. Because the structure of the Uniformed Services Retirement Benefits Act is not unlike that of the PCMC proposal, these conclusions also hold for that plan.

Appendix  
SUPPORTING TABLES

Table A.1  
 STATE DESCRIPTIONS

State Number	Grade	Component	Promotion Group
1	Captain	Reserve	
2	Captain	Regular	
3	Major	Reserve	4
4	Major	Reserve	3
5	Major	Reserve	2
6	Major	Reserve	1
7	Major	Regular	4
8	Major	Regular	3
9	Major	Regular	2
10	Major	Regular	1
11	Lieutenant Colonel	Reserve	4
12	Lieutenant Colonel	Reserve	3
13	Lieutenant Colonel	Reserve	2
14	Lieutenant Colonel	Reserve	1
15	Lieutenant Colonel	Regular	4
16	Lieutenant Colonel	Regular	3
17	Lieutenant Colonel	Regular	2
18	Lieutenant Colonel	Regular	1
19	Colonel	Reserve	4
20	Colonel	Reserve	3
21	Colonel	Reserve	2
22	Colonel	Reserve	1
23	Colonel	Regular	4
24	Colonel	Regular	3
25	Colonel	Regular	2
26	Colonel	Regular	1
27	The Civilian State		

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Table A.2

YEAR OF SERVICE AGGREGATIONS  
FOR PROMOTION GROUPS

Grade/Promotion Group	Years of Service
Major	
1	8-10
2	11-12
3	13-15
4	16-17
Lieutenant Colonel	
1	11-13
2	14-16
3	17-18
4	19-24
Colonel	
1	13-16
2	17-21
3	22-23
4	24-29

REFERENCES

- Chipman, Mark D. and Hans Mumm, "Forecasting Naval Enlisted Retention Behavior Under Alternative Retirement Systems," Navy Personnel Research and Development Center, San Diego, NPRDC TR 79-4, November 1978.
- Diamond, P. and E. Maskin, "An Equilibrium Analysis of Search and Breach of Contract, I: Steady States," *Bell Journal of Economics* 10 (1979), 282-316.
- Dreze, J. H. and F. Modigliani, "Consumption Decisions Under Uncertainty," *Journal of Economic Theory* 5 (1972), 308-335.
- Enns, John H., *Reenlistment Bonuses and First-Term Retention*, The Rand Corporation, R-1935-ARPA, September 1977.
- Gotz, G. A., "A Dynamic Model of Junior Officer Retention Decisions," The Rand Corporation, Corporation, (forthcoming).
- Gotz, G. A. and J. J. McCall, "A Sequential Analysis of the Air Force Officers' Retirement Decision," The Rand Corporation, N-1013-AF, October 1979.
- Heckman, J. J., "Statistical Model for Discrete Panel Data," Draft Paper, University of Chicago, 1978a.
- , "Heterogeneity and State Dependence in Dynamic Models of Labor Supply," Draft Paper, University of Chicago, 1978b.
- Heckman, J. J. and R. J. Willis, "Estimation of a Stochastic Model of Reproduction: An Econometric Approach," in *Household Production and Consumption*, N.E. Terleckyj (ed.), National Bureau of Economic Research, 1976, 99-138.
- Kiefer, Nicholas M. and George R. Neumann, "Individual Effects in a Non-Linear Model: Explicit Treatment of Heterogeneity in the Empirical Job-Search Model," University of Chicago, Report 7921, June 1979.
- Lippman, S. A. and J. J. McCall, "The Economics of Job Search: A Survey," *Economic Inquiry* 14 (1976), 155-189.
- , "The Economics of Uncertainty: Selected Topics and Probabilistic Methods," *Handbook of Mathematical Economics*, K. Arrow and M. Intriligator (eds.), North-Holland, 1980.
- Mortensen, D. T., "Specific Capital, Bargaining, and Labor Turnover," *Bell Journal of Economics* 9 (1978), 572-586.



*Report of the President's Commission on Military Compensation*, Washington, D.C., April 1978.

Warner, John T., "Alternative Military Retirement Systems: Their Effects on Enlisted Retention," Draft Paper, Center for Naval Analyses, December 1978.

Wilburn, Robert C., "The Supply of Military Manpower: The Impact of Income, the Draft and Other Factors on the Retention of Air Force Enlisted Men," Ph.D. Dissertation, Princeton University, 1970.

